THE POSSIBILITY OF AUTOMATICALLY OBTAINING STATISTICAL MORPHOLOGICAL CHARACTERISTICS OF SECTIONS OF THE LUNAR SURFACE FROM THEIR PHOTOGRAPHIC IMAGES

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ABSTRACT. The problem is formulated mathematically, micro- and macrorelief are determined, possibility is discussed of reproducing relief of a region from its photographic image and obtaining statistical morphological characteristics from this relief. Practical ways of realizing this possibility are analyzed. A possibility is shown of obtaining some statistical morphological characteristics of a region by analogy methods without reproducing its relief using coherent light beam technique or correlation methods. Though a theoretical study of the problems is carried out for the Moon, these results with few exceptions are applicable to other planets, such as the Earth, Mars, Mercury as well.

1. Statement of the Problem

The problem of how to determine the morphological characteristics of sections of the lunar surface will not become completely scientific in nature until it is formulated in qualitative concepts that are strictly mathematical in nature. /63

The first step in solving this problem therefore, is the introduction of the mathematical magnitudes that will be used to describe the surface under investigation, and which must be determined from experimental data.

The question of an adequate description of random surfaces similar to the lunar surface is an extremely complex one (although there are no particular difficulties involved in the case of an agitated sea surface, for example), and this paper will make no attempt to respond to it in full. Instead, a simpler approach is selected, even though not the ideal one, at least from the standpoint of complete correspondence with the real object. It is assumed that a section of the lunar surface can be described by the equation

$$H = \varrho(x, y),$$

where H is the altitude, read from some mean, adequately smooth surface determining the figure of the Moon. This surface can be considered to be a /64

plane within the limits of the section under consideration, and x and y are Cartesian coordinates on this plane. The function $\rho(x, y)$ is a discontinuous function, with the measure of the set of points of the discontinuity equal to zero and integrated over any finite region.

This description is too detailed for practical purposes, so it is convenient to represent $\rho(x,y)$ in the form of the sum of two factors

$$\rho(x, y) = R(x, y) + r(x, y)$$

where $R(x, y)$ characterizes the averaged appearance of the surface, and is obtained by smoothing the function $\rho(x, y)$

$$R(x, y) = \int \rho(x', y')\, g(x - x', y - y')dx'dy'$$

for the entire

section

$g(\xi, \eta)$ is a function significantly different from zero only in the region with characteristic size t such that the value of the function $R(x, y)$ at some point is the result of averaging the function $\rho(x, y)$ for the vicinity of this point with characteristic size t.

Thus, the true relief, $\rho(x, y)$, is broken down into the macrorelief $R(x, y)$ and the microrelief $r(x, y)$ so as to characterize the microrelief by mathematical characteristics, retaining the detailed description for the macrorelief only, or so as to introduce suitable statistical characteristics for the macrorelief as well.

If judicious limitations are imposed on the smoothing function, $g(x, y)$, the appearance of the macrorelief at a scale well in excess of t, and the appearance of the microrelief at a scale well below t, will not depend on the concrete form of $g(\xi, \eta)$, which therefore can be selected solely for convenience. Nevertheless, this breakdown into macro- and microrelief is conditional because it contains an arbitrary magnitude, t, the selection of which is beyond the scope of purely mathematical considerations, and should be made with the concrete goals of the research in mind.

The characteristics of the section of the lunar surface, the macrorelief, and the statistical properties of the microrelief, should include its morphological characteristics.

This is a practically acceptable description of a section of the lunar surface, but can be too detailed, depending on the goals of the research. We may not need a detailed description of the macrorelief, R(x, y), to solve a concrete problem, for example, but instead require its local properties, or, on the other hand, certain integral properties, statistical characteristics in particular. The selection of these latter is entirely dependent on the goals of the research and can be made only after the concrete questions to which answers must be provided during the research have been formulated.

This article is devoted to a preliminary explanation of the possibilities, in principle and in practice, of using photography to obtain the macrorelief, R(x, y), and certain integral characteristics of that relief, that can be useful in solving scientific and technical problems.

2. Restoration of the Relief of a Section of the Lunar Surface from Its Photographic Image.

The apparent brightness of a small section of the lunar surface, which can be taken as approximately flat, depends on its orientation when the positions of Sun and observer are fixed. The optical density of the photographic image /65 therefore contains information at each point as to the magnitude of the slope of the surface at the point. And because the slope is a differential characteristic of the relief, its image provides information on the relief. The completeness and reliability of this information depends entirely on the quality and quantity of original photographic data. Diggelen [1] was first to suggest photometric means for restoring the relief, and further development took place in the work of the Main Astronomical Observatory of the Academy of Sciences of the Ukrainian SSR [2] for lunar application.

Let us review the relationships on which the possibility of solving this problem, in principle, is based.

Let us say we have the image of a small section of the lunar surface, flat on the average, with photometric longitude λ_0 and photometric latitude φ_0. Let

us overlay this section with a Cartesian system of coordinates, x, y, such that the x-axis is oriented along the photometric parallel, and the y-axis along the photometric meridian. If the section under consideration is small enough it can be taken that the photometric coordinates of an infinitely small area of this section have the values λ and φ, which depend solely on the slope of this area, but are not explicitly dependent on its Cartesian coordinates.

Let the photometric properties of the section studied be identical at all points on the surface, and the dependence of the brightness of the area on the lunar surface, I, on the photometric coordinates λ and φ be characterized by the function $B(\lambda, \varphi, \alpha)$, where α is the phase angle for the given position of the Sun, and for the observer, such that

$$I = B(\lambda, \varphi, \alpha) \tag{2.1}$$

Then brightness in terms of x, y can be represented in the following form because of the dependence of the slope on x, y for small $\Delta\lambda = \lambda - \lambda_0$ and $\Delta\varphi = \varphi - \varphi_0$, with an accuracy to magnitudes of the order $(\Delta\lambda)^2$ and $(\Delta\varphi)^2$

$$I(x, y) = B(\lambda_0, \varphi_0, \alpha) + \frac{\partial B(\lambda, \varphi, \alpha)}{\partial \lambda}\Big|_{\substack{\lambda=\lambda_0 \\ \varphi=\varphi_0}} \cdot \Delta\lambda(x, y) + \frac{\partial B(\lambda, \varphi, \alpha)}{\partial \varphi}\Big|_{\substack{\lambda=\lambda_0 \\ \varphi=\varphi_0}} \Delta\varphi(x, y). \tag{2.2}$$

Now, in order to link brightness in terms of the coordinates I(x, y) with the macrorelief of the section, R(x, y), let us take advantage of the fact that

$$\Delta\lambda = -\operatorname{arc\,tg} \frac{\partial R(x, y)}{\partial x}; \quad \Delta\varphi = -\operatorname{arc\,tg} \frac{\partial R(x, y)}{\partial y}, \tag{2.3}$$

or, for small $\Delta\lambda$ and $\Delta\varphi$, approximately

$$\Delta\lambda = -\frac{\partial R(x, y)}{\partial x}; \quad \Delta\varphi = -\frac{\partial R(x, y)}{\partial y}, \tag{2.4}$$

so that Eqs. (2.2) and (2.4) will yield

$$I(x, y) = I_0 - \frac{\partial B(\lambda, \varphi, \alpha)}{\partial \lambda}\Big|_{\substack{\lambda=\lambda_0 \\ \varphi=\varphi_0}} \cdot \frac{\partial R(x, y)}{\partial x} - \frac{\partial B(\lambda, \varphi, \alpha)}{\partial \varphi}\Big|_{\substack{\lambda=\lambda_0 \\ \varphi=\varphi_0}} \frac{\partial R(x, y)}{\partial y} \tag{2.5}$$

where

$I_0 = B(\lambda_0, \varphi_0, \alpha)$, or

$$J(x, y) = b_x \frac{\partial R(x, y)}{\partial x} + b_y \frac{\partial R(x, y)}{\partial y}, \tag{2.6}$$

where

$$J(x, y) = 1 - \frac{J(x, y)}{J_0},$$

$$b_x = \frac{1}{J_0} \cdot \frac{\partial B(\lambda, \varphi; \alpha)}{\partial \lambda}\Big|_{\substack{\lambda=\lambda_0 \\ \varphi=\varphi_0}}, \quad b_y = \frac{1}{J_0} \cdot \frac{\partial B(\lambda, \varphi, \alpha)}{\partial \varphi}\Big|_{\substack{\lambda=\lambda_0 \\ \varphi=\varphi_0}}. \tag{2.7}$$

Eq. (2.6) is a linear partial differential equation that enables us to find R(x, y), knowing the dependence on the coordinates of the relative brightness J(x, y). But Eq. (2.6) must be supplemented by boundary conditions to arrive at an unambiguous localization of R(x, y). The most natural thing to do is to assign the R(x, y) value along some contour in the lunar section investigated that is not a characteristic line of this equation. /66

Numerous observations show [2, 3] that Eq. (2.1) for the lunar surface is such that $b_y = 0$ always is so and that Eq. (2.6) is converted into an ordinary differential equation when the system of coordinates has been selected as above

$$\frac{dR}{dx} = \frac{1}{b_x} J(x, y) \tag{2.8}$$

the solution of which is

$$R(x, y) = R_0(y) + \frac{1}{b_x} \int_0^x J(x', y)\, dx' \tag{2.9}$$

and contains the constant of integration $R_0(y)$. Here the characteristic lines of Eq. (2.6) are straight lines, parallel to the x-axis, and the unambiguous localization of the relief, R(x, y) is possible only in a band parallel to the x-axis and intersecting the contour to which the boundary condition was assigned.

This boundary condition usually is unknown, however. In such cases it can be obtained (within a constant factor) from other photography, obtained under different illumination conditions, when the photometric meridian is in another direction. It is impossible to make an unambiguous determination of the relief if the second photographs are not available to the researcher. Now other information on the nature of the region studied can be used as the basis for plausible assumptions as to its statistical qualities, and these qualities then can be used to find the most plausible form of R(x, y), with consideration given to Eq. (2.8), or Eq. (2.9). There is a broad class of cases when this approach leads to the conclusion that the most plausible relief, R(x, y), satisfies the condition

$$\int R(x, y)dx = 0 \qquad (2.10)$$

within the limits of the region. It replaces the boundary condition and, together with Eq. (2.9), uniquely determines the function R(x, y), which now should be understood to be the most probable, but completely optional, true macrorelief.

This simple arrangement for finding the relief from the image applies only to the case when the image contains no shadows. Ordinarily, however, illumination adequate for discovering the relief forms shadows. It is easy to take this fact into consideration, but in such case only the relief of the illuminated sections can be identified, naturally. By way of illustration, let us consider the simplest, yet extremely common, case of shadows falling along the photometric parallel. This occurs, in particular, when the region studied is in the center of the disk, that is, when the observer views it along a normal to its mean plane.

Let a straight line, parallel to the y-axis, intersect the boundary of the shadows at points x_0, x_1, x_2 ...x_n, and when k is even, x values satisfying the inequality

$$x_k < x < x_{k+1}, \qquad (2.11)$$

correspond to the illuminated points.

Then relief R(x, y) satisfies Eq. (28) [sic] at each of these intervals, remains ambiguous in the intervals

$$x_{k+1} < x < x_{k+2} \qquad (2.12)$$

when k is even. We can relate the R(x, y) values in the different intervals of (2.11) by taking advantage of the fact that the point (x_{k+2}, y) (for even k) is the end of the shadow cast by point (x_k, y) such that

$$R(x_{k+2}, y) - R(x_{k+1}, y) = (x_{k+2} - x_{k+1}) \operatorname{ctg} i, \qquad (2.13)$$

where i is the angle of incidence of the light at the lunar region under study (if the shadow is cast along the x-axis in the positive direction). The angle i is readily reflected in photometric coordinates.

Eqs. (2.8) and (2.13) together form a differential-difference equation, the solution of which in the interval (x_{k+2}, x_{k+3}) can be expressed recursively by a solution in the interval (x_k, x_{k+1}) of the equation

$$R(x, y)=R(x_{k+1}, y)+\frac{1}{b_x}\int_{x_{k+2}}^{x} R(x', y)\,dx'-(x_{k+2}-x_{k+1})\operatorname{ctg} i. \tag{2.14}$$

This solution contains an arbitrary constant, $R(x_0, y)$, quite analogous to the constant $R_0(y)$ in Eq. (2.8). Everything said about $R_0(y)$ applies as well to $R(x_0, y)$. It should be noted, in passing, that analysis of the form of the shadow boundaries could yield additional information about the relief. This question is beyond the scope of this article, however.

The results of the review cited in this section are tentative in nature because Eq. (2.2) was obtained from the approximate replacement of the function $B(\lambda, \varphi, \alpha)$ by the linear part of a Taylor series, and is applicable only to small slopes. This simplification in no way concerns the principle involved, but its rejection will make it necessary to solve a more complex equation than Eq. (2.6).

On the other hand, the effect on the results obtained of random errors, unavoidable in all measurements, has not been included here. Consideration of these errors can disclose additional limitations on the practical possibility of finding the relief from the photometric image.

Finally, this review is based on the assumption of photometric homogeneity of the section studied, something that seldom is the case in fact. Consideration of photometric heterogeneity of a region can require a third round of photographs, if it is necessary to find a unique solution for $R(x, y)$, or additional information of a statistical nature if one is restricted to discovering the statistically plausible form of $R(x, y)$.

Thus, the question reviewed in this section requires additional study, primarily involving expanding the region of applicability of the theory, evaluating the accuracy of the results obtained, seeking the optimum criteria for plausibility when data are inadequate, optimum methods of taking excess information (if available) into consideration, as well as along the lines of a more detailed theoretical and experimental investigation of the law of the reflection of light from the lunar surface.

3. Analog Methods for Obtaining the Correlation Function for the Macrorelief from the Photographic Image of the Region

This section will discuss methods of obtaining the correlation function for altitudes by analog means /68

$$K_R(\xi, \eta) = M[R(x, y) \cdot R(x + \xi, y + \eta)], \tag{3.1}$$

based on its connection with the correlation function for brightness, $C(\xi, \eta)$, determined by the equality

$$C(\xi, \eta) = M[J(x, y) \cdot J(x + \xi, y + \eta)], \tag{3.2}$$

where M is the mathematical expectation. The surface is assumed statistically homogeneous. This connection can be obtained if the relative brightness $J(x, y)$, is expressed in terms of the slope using Eq. (2.6), if $b_y = 0$ for the Moon [3, 4], and if the expression obtained can be substituted into Eq. (3.2). This leads to

$$C(\xi, \eta) = b_x^2 M\left[\frac{\partial R(x, y)}{\partial x} \cdot \frac{\partial R(x + \xi, y + \eta)}{\partial x}\right] \tag{3.3}$$

The following equality can be shown to exist for the mathematical expectation in the right-hand side of Eq. (3.3) for a section characterized by the correlation function for altitudes, $K_R(\xi, \eta)$

$$M\left[\frac{\partial R(x, y)}{\partial x} \cdot \frac{\partial R(x + \xi, y + \eta)}{\partial x}\right] = -\frac{\partial^2 K_R(\xi, \eta)}{\partial \xi^2}, \tag{3.4}$$

such that the unknown relationship has the form

$$\frac{\partial^2 K_R(\xi, \eta)}{\partial \xi^2} = -\frac{1}{b_x^2} C(\xi, \eta), \tag{3.5}$$

and is an ordinary differential equation for $K_R(\xi, \eta)$. The boundary condition for it, permitting the finding of a unique solution, follows from the general properties of the autocorrelation function, evenness, and vanishing at infinity, and has the form

$$K_R(\infty, \eta) = 0, \; K_R(\xi, \eta) = K_R(-\xi, \eta). \tag{3.6}$$

Its solution, boundary conditions considered, has the form

$$K_R(\xi, \eta) = -\frac{1}{b^2}\int\int C(\xi'', \eta)\, d\xi''\, d\xi'. \tag{3.7}$$

So the autocorrelation function for Eq. (3.1) altitudes can be found without image restoration of relief R(x, y) if the Eq. (3.2) correlation function is found by the analog method for the image (this requires an image with a contrast factor of unity).

Two methods are available for determining the Eq. (3.2) autocorrelation function. One, let us call it the correlation method, involves measurement of the integral transparency of a pair of photographic plates with images of the section under investigation so composed that the integral transparency is equal to the product of the brightness in Eq. (3.2) within a constant summand sign and factor. The Eq. (3.2) correlation function, can, for example, be calculated by using the following formula, based on the ergodicity of the realization of the ensemble of random surfaces, /69

$$C(\xi, \eta) \approx \frac{1}{S}\int J(x, y)\cdot J(x+\xi, y+\eta_1)\, dx\, dy,$$

for entire region (3.8)

is obtained as the dependence of the integral transparency of this pair of plates on the magnitude of the shift between them [5]. Here S is the area of the region investigated.

The second method can be called the spectral method because it involves finding the Eq. (3.2) correlation function in terms of the spatial spectrum of the photographic image on the basis of the Weiner-Khinchin theorem

$$C(\xi, \eta) = \int_{-\infty}^{\infty}\int S(\mu, \nu)\, e^{i(\mu\xi+\nu\eta)}\, d\mu\, d\nu, \tag{3.9}$$

where $S(\mu, \nu)$ is the spatial spectrum of the photographic image (with contrast equal to unity) of the region investigated.

There is a simple analog method for obtaining the spectrum of the photographic image that uses the coherent bundles technique [6]. It is based on the fact that the optical image in the focal plane of the objective lens is a

Fourier transform of the amplitude of the light wave in the plane of the objective lens aperture. This spectrum is obtained in the form of blackenings on the photographic plate which then must be subjected to photometry and used to make the Eq. (3.9) calculation.

4. The Possibility of Obtaining Crater Distribution by Size from the Brightness Correlation Function

The analog methods reviewed in the preceding section can, in principle, be used to obtain finer statistical characteristics of the region investigated. This possibility can be illustrated by the example of the distribution of craters by size for the ideal case when there are no other types of formation in a specified region, and all craters are geometrically similar and differ only in size.

Let the distribution of crater brightness be described by the function $i\left(\frac{x - x_0}{a}; \frac{y - y_0}{a}\right)$, where a is the crater diameter and x_0 and y_0 are the coordinates of its center. Then, in the case of uniform displacements along the x-axis, the correlation function for the image of this crater

$$k(\xi, a) = \int_{-\infty}^{\infty}\int i\left(\frac{x}{a}, \frac{y}{a}\right) i\left(\frac{x+\xi}{a}, \frac{y+\eta}{a}\right) dx\, dy \tag{4.1}$$

can be represented in the form

$$k(\xi, a) = a^2 k_0\left(\frac{\xi}{a}\right), \tag{4.2}$$

where

$$k_0(\xi) = k_0(\xi, 1). \tag{4.3}$$

If it is assumed that the density of craters in the area considered is sufficiently low, such that the characteristic distances between them are very much greater than the characteristic sizes, the one-dimensional correlation function $C(\xi)$ for the entire image, found using Eq. (3.2) when $\eta = 0$, is equal to /70

$$C(\xi) = \int_0^\infty k(\xi, a) f(a)\, da, \tag{4.4}$$

where f(a) is the unknown crater size distribution function.

$C(\xi)$ can be represented in the following form, using Eq. (4.2)

$$C(\xi) = \int_0^\infty k_0\left(\frac{\xi}{a}\right) f(a)\, a^2\, da. \tag{4.5}$$

This is an integral equation linking the correlation function for the brightness with the function for the distribution of craters by size. It can be reduced to an integral equation with a difference kernel if a conversion is made from variables ξ, a to the variable

$$X = \ln \xi,\ A = \ln a. \tag{4.6}$$

Then Eq. (6.5) [sic] becomes

$$\int_{-\infty}^{\infty} G(X-A)\cdot F(A)\, dA = Z(X),$$

where

$$G(X) = k_0(e^X),\ F(X) = e^{2X} f(e^X),\quad Z(X) = C(e^X). \tag{4.7}$$

There is no difficulty in finding the solution of Eq. (4.7), if it exists, by using the Fourier method.

The considerations set forth show that it is possible, in principle, but by no means possible in practice, to solve the problem posed because not taken into consideration here is the effect of measurement errors which, in problems of this type, usually are decisive. The question of the practical possibility of solving this problem requires separate investigation.

Conclusion

This article has considered ways to obtain the correlation function for the relief of a region under investigation from its photographic image by using analog type devices.

We have not considered more effective ways to find the statistical and morphological characteristics, accomplishment of which requires the use of electronic digital computers. The most universal method of automating the process of obtaining the statistical and morphological characteristics in this case is to put the image into a computer and obtain an algorithm for recognizing the objects, after which their classification (and comparison) and inclusion using the computer involves no special difficulties, in principle.

However, the object recognition problem is one of the most complex problems of modern cybernetics. It can be solved quite readily as a problem of making a statistical check of a set of hypotheses when posed in abstract form and as applicable to an individual model. The answer lies in finding an object that corresponds to the model with maximum a posteriori probability, but this means calculating the a posteriori probability for every possible object, using Bayes formula, and finding the object to which the maximum probability corresponds. /71 This problem is just as simple, and just as complex, in practice as it is in principle when this approach is taken, although there is a broad class of problems for which this approach can be used, readily, and easily. In the majority of the complex problems it is impossible to apply the a posteriori probability calculated for one object, assuming this can be done, to all objects. Hence the possibility of solving the problem using an optimum formulated algorithm remains a purely abstract possibility that will not lead to a practical solution of the problem. This problem now must be formulated as one of finding a practically realizable recognition algorithm which, if not optimum, is as close to the optimum in effectiveness as possible. The problem considered here is further complicated by the fact that models subject to recognition are not individualized because they have boundaries that are known in advance.

Still, the complexity of the problem considered ought not act as a deterrent to every effort to solve it by using this approach. The first step toward its solution can be a transition from the photographic image of the section investigated to its relief (#2). This eliminates a multiplicity of models corresponding to the same object (because it can be illuminated differently) and significantly simplifies the formulation of criteria distinguishing an object of a particular class from objects of another class. Each of these circumstances can simplify the recognition problem significantly. The complexity and time required to develop this method will pay for itself in full in its great possibilities and high degree of operability in solving newly posed problems.

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